Calculations concerning voltage ripple of x-ray generators

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Abstract. Formulae are given for absorbed dose, fluence and energy fluence of narrow x-ray beams generated by pulsating-potential x-ray generators with any waveform. A computer program has been written for calculating these and some other quantities, including mean energies, for particular waveforms. A new definition is suggested for the ripple correction factor of pulsating-potential x-ray generators for the assessment of their performance. Some numerical results and applications are discussed.

1. Introduction

Although rectifying systems different from the conventional 1-, 2-, 6- and 12-pulse types have existed for a long time, only the increasing availability of inverter (i.e. medium and industrial frequency) x-ray generators has produced a serious demand for a physically based comparative assessment of all these generator types (IEC †). This assessment must cover the comparison of both radiation quantity (i.e. fluence, energy fluence and dose) and radiation quality. So far, there have been only two standardised quantities in the publications of the International Electrotechnical Commission (IEC) concerning x-ray tube voltage waveforms. One of them is percentage ripple, which is defined as $R = (U_{\text{max}} - U_{\text{min}})/U_{\text{max}}$ (as a percentage) where $U_{\text{max}}$ and $U_{\text{min}}$ are the maximum and minimum values of the x-ray tube voltage, respectively. The other quantity is the factor $f$ (it has no standardised term), which is fixed at discrete values: $f = 0.74$ for 1- and 2-pulse, $f = 0.95$ for 6-pulse and $f = 1.0$ for 12-pulse and constant potential x-ray generators.

According to earlier documents of the IEC (1979) one of these three values must also be applied to any other rectifying system. It is obvious that for a correct assessment of the performance of new rectifying systems these parameters must be superseded.

Our goal is therefore to give general formulae that give explicitly the values of fluence, energy fluence, and absorbed dose in air and in water and which specify x-ray qualities, mean energies for narrow x-ray beams generated by pulsating-potential x-ray generators with any waveform, attenuated by any medium, and to calculate them practically—at least in a first approximation. From these considerations, it seems to be possible to choose the most adequately defined quantity for the assessment of x-ray generator performance. Such a quantity can be useful for the assessment not only of newer x-ray generators but also of older types. In addition, the method has some other applications. Preliminary reports on this work have been given earlier (Porubszky 1983, 1985).

2. General remarks

An ideal x-ray generator is always considered to be a constant potential one, because of the increase of softer x-ray components in the case of any pulsation. (Although the proportion of softer rays can be diminished by filtration, dose rates will then be smaller, because of the limited x-ray tube loadability; moreover, an x-ray spectrum identical with that of a constant potential generator cannot be reached by filtration — see figure 3 below.)

The advantage of medium frequency x-ray generators is that rapidly and precisely controlled short exposures, satisfying the demands of digital image storage, can be performed even with single-phase mains; furthermore, because of its frequency, the x-ray tube voltage can be smoothed more effectively and it can closely approximate the constant potential.

Deviation from constant potential depends upon frequency, electrical smoothing, tube loading etc. An improper design or adjustment can cause — in the case of higher tube loads — significant ‘spikes’ in the tube voltage and current waveforms. This is illustrated in figure 1 with an actual oscilloscope photograph. It can be seen that the value of percentage ripple $R$ is about 40%, while there is a feeling that the difference of this tube voltage from a constant potential — from the point of view of radiation output — is significantly smaller than 40%. This statement is to be given in a quantitative form.

† IEC 1984 Unconfirmed Minutes of Meeting SC62B, Maastricht 1984 (Secretariat IEC TC62) p 5 (point V.4) and p 15 (point XXII.3: ‘. . . the meeting decided . . . to consider particular aspects of converter generators as a next new item of work’. The proposal was offered by Mr J Barsai of Medicor, Hungary).
In specifying x-ray techniques, tube voltages are conventionally given in peak values while tube currents as their mean values—mostly for reasons resulting from measuring techniques. From the point of view of tube loading, however, the maximum values are important only because maxima of voltage and current always coincide. (This is the reason why x-ray tubes must be loaded by lower mean currents in the 2-pulse mode than in 6- or 12-pulse modes.) In our opinion, the comparison of x-ray outputs is appropriate only for the same current maxima, while usually mean values are given and this by-passes differences among x-ray generators.

3. Preliminaries

Effects of voltage waveforms (on dose, radiation quality parameters, and spectra) were investigated experimentally by many authors. A comprehensive theoretical study, however, has not been made so far.

McCullough and Cameron (1970) approximated exposure output ratios of conventional x-ray generator types with constant values irrespective of the tube voltage. Birch et al (1976) elaborated a spectrometer system synchronised by the mains voltage cycle and they measured instantaneous spectra within the period time. A formula for time-averaging of instantaneous spectra was first stated by Hettinger and Starfelt (1958) who have also computed it numerically for a given type of waveform. Applying a similar expression, O'Foghludha and Johnson (1981) introduced correction factors expressing the proportion of the exposure output of a given generator relative to a constant potential one. Unfortunately, they approximate the constant potential exposure output by a power function of the tube voltage. It cannot be considered as a general description because the value of the exponent can only be determined experimentally, no general value, even approximately, can be given to it.

Most of the authors calculating the attenuation and absorbed dose of heterogeneous x-ray beams have restricted their work to the case of constant potential. There were only a few attempts to take into account the effects of pulsation with the aid of weighted averaging of spectra approximated by Kramers’ formula. Stanton et al (1979) have fitted Kramers’ spectra by empirical factors to measured values before averaging. Birch et al (1979) presented pulsating-potential x-ray spectra calculated by averaging theoretical spectra (Birch and Marshall 1979) fitted to some experimental data, and also included the attenuation due to inherent filtration and air path.

4. Formulae to be calculated

Taking into account some known expressions (i.e. mass attenuation coefficients of mixtures, absorbed dose from a heterogeneous beam, averaging of time dependent quantities) and the preliminaries mentioned above, we obtain the following formula for the absorbed dose from narrow x-ray beams generated by pulsating-potential x-ray generators with any waveform, passed through any attenuating medium (Porubszky 1983):
\[ D(x) = \frac{t}{Tx^2} \int_0^{E_{\text{max}}} A(E)(\mu_{\text{en}}/\rho)(E, x)EB(E)dE \]  

(1)

where

\[ A(E) = \exp\left[-\int_{x_0}^x \sum_{i=1}^{n(x')} w_i(x')(\mu/\rho)_i(E)\rho(x')dx'\right] \]  

(2)

and

\[ B(E) = \int T_0^T \phi_0(U(t'), E)i(t')dt' \]  

(3)

where \( x \) is the space coordinate, \( x = 0 \) is the point of origin of the x-rays (tube focus), \( x_0 \) is the position of the tube housing window, \( t' \) is the time, \( t \) is the duration of the irradiation, \( T \) is the period time of the tube voltage, \( i \) is the tube current, \( U \) is the tube voltage, \( E \) is the (photon) energy, \( \phi_0 \) is the spectral distribution of (photon) fluence rate with respect to energy normalised to unit \( i \) and \( x \), \( \rho \) is the density, \( \mu/\rho \) is the mass attenuation coefficient, \( \mu_{\text{en}}/\rho \) is the mass energy absorption coefficient and \( D \) is the absorbed dose. The quantity in square brackets represents the mass attenuation coefficient of an \( n \)-component compound or mixture in which \( w_i \) is the fraction by weight of the \( i \)th component and \( (\mu/\rho)_i \) is its mass attenuation coefficient. \( A(E) \) expresses the total transmitted fraction of photons of the beam passed through all the attenuating media while \( B(E) \) expresses the average fluence rate spectrum normalised to unit maximum x-ray tube current and unit distance from the tube focus without attenuation. In equation (3) \( \phi(E) \) is used instead of \( \phi_0 \) which was standardised in ICRU Report 33 for spectral fluence. The inherent filtration of the x-ray tube is included in the \( \phi_0 \) spectrum while added filters are to be taken into account among the attenuating media.

Based on equations (1), (2) and (3), the fluence can be written in the following form:

\[ \Phi(x) = \frac{t}{Tx^2} \int_0^{E_{\text{max}}} A(E)B(E)dE \]  

(4)

while the energy fluence is

\[ \Psi(x) = \frac{t}{Tx^2} \int_0^{E_{\text{max}}} A(E)EB(E)dE. \]  

(5)

(Equations (1), (4) and (5) do not include the contribution of scattered radiation (narrow-beam geometry) as this contribution cannot be expressed in such a direct form but can be estimated using some approximation methods.)

Fluence, energy fluence and absorbed dose in air and in water characterise the quantity of x-rays. To compare x-ray generator types it is also necessary to give some radiation quality parameter. In our opinion, the most suitable one is the mean energy, which is not only a very descriptive quantity but can also be calculated easily. Its definition is

\[ \bar{E} = \frac{\int_0^{E_{\text{max}}} K(E)E \, dE}{\int_0^{E_{\text{max}}} K(E) \, dE} \]  

(6)

where \( K(E) \) can be the spectral distribution of fluence, energy fluence or exposure.

5. Ripple correction factor for x-ray generators

To characterise the radiation output of x-ray generators, the concept of percentage ripple must be abandoned and a ripple correction factor must be defined which characterises the actual performance of a generator relative to a constant potential one, and which can be calculated for any waveform. It is best given in terms of energy fluence as it is constructed for the evaluation of radiation quantity, useful for image formation rather than patient dose. Thus we can give the following definition for the ripple correction factor:
\[ F = \frac{\int_{E_0}^{E_{\text{max}}} E \phi(E) \, dE}{\int_{E_0}^{E_{\text{max}}} i(U_{\text{max}}) T \left( \int_{E_0}^{E_{\text{max}}} E \phi_0(U_{\text{max}}, E) \, dE \right)} \]

where \( U_{\text{max}} \) is the peak kilovoltage, \( E_0 \) is the lower limit of photon energy due to inherent filtration (e.g. 10 keV), the other symbols are the same as before. The effect of the standardised minimum filtration is included in the \( \phi_0 \) spectrum. It might be useful for the values of \( U_{\text{max}} \) filtration, and other radiographic parameters as well as \( E_0 \) to be fixed as a standard. We suggest giving this \( F \) factor and a mean energy together for comparative evaluation of x-ray generator types.

6. The computer program and sources of data

We have rewritten equations (1) to (7) in a form applicable for numerical calculation and have written a computer program to carry this out.

Voltage and current waveforms, as a first approximation, are idealised, i.e. approximated with analytical functions. Waveforms used so far are presented in table 1. (The choices of the approximations of current waveforms were based on practical experience.) For 6- and 12-pulse x-ray generators a constant current approximation similar to that of Birch et al (1979) was also used. Considering the heating current to be constant, a power function fitted to the x-ray tube emission characteristics of the form

\[ i(t) \sim U^a(t) \]

seems to be the best current approximation for us. It was applied for the calculation of 6- and 12-pulse generators as an option and for 2-pulse generators as the only current waveform. (A weighting similar to expression (8) was also performed by Hettinger and Starfelt (1958).) Inverter-type waveforms were chosen to have 40, 20, 10 and 5% spike depths for voltage, and 75% of them for the corresponding currents while the width of spikes was 25% of the period time.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type</th>
<th>Tube voltage</th>
<th>Tube current</th>
<th>Time interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2-pulse</td>
<td>( U(t) = U_{\text{max}} \sin \frac{2\pi t}{T} ) ( i(t) = U^a(t) )</td>
<td>( 0 \leq t &lt; \frac{T}{4} )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6-pulse</td>
<td>( U(t) = U_{\text{max}} \sin 2\pi \left( \frac{T}{6} + \frac{1}{6} \right) ) ( i(t) = \frac{i_{\text{max}}}{2} \left( 1 + \sin 2\pi \left( \frac{2t}{T} + \frac{1}{12} \right) \right) )</td>
<td>( 0 \leq t &lt; \frac{T}{12} )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12-pulse</td>
<td>( U(t) = U_{\text{max}} \cos \frac{2\pi t}{T} ) ( i(t) = \frac{i_{\text{max}}}{2} \left( 1 + \cos \frac{4\pi t}{T} \right) )</td>
<td>( 0 \leq t &lt; \frac{T}{24} )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>40% spike</td>
<td>( U(t) = U_{\text{max}} \left( C_1 \frac{t}{T} + C_2 \right) ) ( i(t) = i_{\text{max}} \left( C_1 \frac{t}{T} + C_4 \right) )</td>
<td>( 0 \leq t &lt; \frac{T}{4} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20% spike</td>
<td>( U(t) = U_{\text{max}} \left( C_1 \frac{t}{T} + C_2 \right) ) ( i(t) = i_{\text{max}} \left( C_1 \frac{t}{T} + C_4 \right) )</td>
<td>( 0 \leq t &lt; \frac{T}{4} )</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10% spike</td>
<td>( U(t) = \frac{U_{\text{max}}}{2} ) ( i(t) = i_{\text{max}} )</td>
<td>( \frac{T}{4} \leq t &lt; T )</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5% spike</td>
<td>( U(t) = U_{\text{max}} ) ( i(t) = i_{\text{max}} )</td>
<td>any</td>
<td></td>
</tr>
</tbody>
</table>

Values of \( \mu / \rho \) were taken from Pletchaty et al (1981) and values of \( \mu_{\text{ad}} / \rho \) for air and water from Hubbell (1977). A linear interpolation was carried out on a log-log scale between the tabulated values.

Most of the spectra were taken from the catalogue of Birch et al (1979) (for 50, 60, 70, 80, 90 and 100 kV constant potential, including the attenuation effect of 2 mm Al equivalent inherent filtration and 75 cm of air), completed by some of our own measured spectra (Porubszky et al 1984, for 26 and 40 kV).

The integration with respect to \( x \) is reduced to the choice of the thicknesses of some attenuating media. (The summation with respect to energy was made in 1 keV steps and with respect to time in steps of \( T/480 \), for the time
intervals given in table 1, where \( T \) is the period time.)

Input data (variables) of the program are: generator (waveform) type (according to table 1), kV\(_p\), Al filtration, focus-to-object distance (attenuation in air is also taken into account), and thicknesses of some other attenuating media. In the different variants of the program these can be a copper filter, a water phantom, soft tissue, bone, the material of the radiographic tabletop, the lead shielding etc.

Output data (results) of the calculations are: fluence, energy fluence and absorbed dose rates (in air and water) normalised to unit (i.e. 1 mA) tube current; the factor \( F \) and the three types of mean energy are also calculated. Radiation quantity data are expressed first for the same (1 mA) tube current maximum, then also for the same (1 mA) current mean. The ratio between them is:

\[
\frac{i_{\text{mean}}}{i_{\text{max}}} = \frac{1}{T} \int_0^T i(t) dt.
\]

(9)

### 7. Results and discussion

Some results for \( U = 100 \) kV\(_p\), are shown in table 2. For the specified beams we have also calculated the energies absorbed in a 250 \( \mu \)m thick CsI detector layer (corresponding to the relative input signal value of an x-ray image intensifier) and the input signal contrast \( C_s = (S_1 - S_2) / (S_1 + S_2) \) was formed from them. It can be seen from the table that the dose rate (and the corresponding radiation quality) of 2-pulse generators is extremely bad. (For a 1-pulse generator the mean energies are equal to those of a 2-pulse generator while radiation quantity data for a 1-pulse generator would be a half of the corresponding 2-pulse values.) It can also be stated that from inverter waveforms, from the point of view of radiation quantity, the 40 and the 10% spike waveforms correspond approximately to 6- and 12-pulse generators, while from the point of view of radiation quality the correspondence is with the 10 and the 5% spike waveforms, respectively.

For a correct choice of a ripple correction factor characteristic for x-ray generators (and also for illustration), we have, in addition to the factor \( F \) defined in § 5, also calculated some other correction factors that may come into question. Thus the electrical power factor \( f_{el} \) is defined as

\[
f_{el} = \frac{1}{T} \int_0^T \frac{U(t) i(t)}{U_{\text{max}} i_{\text{max}}} dt
\]

(10)

and expresses the ratio of effective power to apparent power.

**Table 2.** Some results for \( U = 100 \) kV\(_p\) tube voltage, 2mm Al equivalent inherent filtration and 1 m focus-to-object distance. Generator types: see table 1. Suffixes: 1: analytical; 2: power function; 3: constant current approximation. Rates are normalised to 1 mA current maximum.

<table>
<thead>
<tr>
<th>Type</th>
<th>Attenuator</th>
<th>( \frac{i_{\text{mean}}}{i_{\text{max}}} )</th>
<th>( D_{\text{ex}} / i ) (( \mu \text{Gy} / \text{mAs} ))</th>
<th>( \frac{\varphi_i}{\varphi_i} ) (l/mAs cm(^2))</th>
<th>( \varphi_i ) (l/mAs cm(^2))</th>
<th>( F )</th>
<th>( E_{\text{ex}} ) (keV)</th>
<th>( E_{\text{em}} ) (keV)</th>
<th>( E_{\text{es}} ) (keV)</th>
<th>( D_{\text{ex}} / i ) (( \mu \text{Gy} / \text{mAs} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>—</td>
<td>1</td>
<td>161.6</td>
<td>3.24 ( \times 10^4 )</td>
<td>2.49 ( \times 10^6 )</td>
<td>1</td>
<td>48.0</td>
<td>54.0</td>
<td>40.3</td>
<td>165.7</td>
</tr>
<tr>
<td>8</td>
<td>—</td>
<td>0.082</td>
<td>83.76</td>
<td>1.46 ( \times 10^4 )</td>
<td>1.03 ( \times 10^6 )</td>
<td>0.416</td>
<td>44.1</td>
<td>50.1</td>
<td>35.8</td>
<td>85.60</td>
</tr>
<tr>
<td>1</td>
<td>—</td>
<td>0.913</td>
<td>139.3</td>
<td>2.73 ( \times 10^4 )</td>
<td>2.05 ( \times 10^6 )</td>
<td>0.825</td>
<td>47.0</td>
<td>52.9</td>
<td>39.3</td>
<td>142.6</td>
</tr>
<tr>
<td>1</td>
<td>—</td>
<td>0.982</td>
<td>148.7</td>
<td>2.91 ( \times 10^4 )</td>
<td>2.19 ( \times 10^6 )</td>
<td>0.878</td>
<td>47.0</td>
<td>52.9</td>
<td>39.3</td>
<td>152.3</td>
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<tr>
<td>1</td>
<td>—</td>
<td>1</td>
<td>151.2</td>
<td>2.96 ( \times 10^4 )</td>
<td>2.22 ( \times 10^6 )</td>
<td>0.892</td>
<td>46.9</td>
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<td>154.8</td>
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<td>0.978</td>
<td>154.9</td>
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<td>2.36 ( \times 10^6 )</td>
<td>0.947</td>
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<tr>
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<td>0.995</td>
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<td>53.8</td>
<td>40.0</td>
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</tr>
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<td>1</td>
<td>158.7</td>
<td>3.16 ( \times 10^4 )</td>
<td>2.41 ( \times 10^6 )</td>
<td>0.969</td>
<td>47.7</td>
<td>53.8</td>
<td>40.0</td>
<td>162.6</td>
</tr>
<tr>
<td>1</td>
<td>—</td>
<td>0.963</td>
<td>147.4</td>
<td>2.90 ( \times 10^4 )</td>
<td>2.20 ( \times 10^6 )</td>
<td>0.883</td>
<td>45.1</td>
<td>50.8</td>
<td>37.6</td>
<td>151.0</td>
</tr>
<tr>
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<td>—</td>
<td>0.982</td>
<td>153.9</td>
<td>3.05 ( \times 10^4 )</td>
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<td>52.0</td>
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<td>53.6</td>
<td>39.9</td>
<td>163.6</td>
</tr>
<tr>
<td>2</td>
<td>17 cm soft</td>
<td>0.995</td>
<td>2.129</td>
<td>6.32 ( \times 10^4 )</td>
<td>6.10 ( \times 10^8 )</td>
<td>0.965</td>
<td>60.3</td>
<td>64.0</td>
<td>58.3</td>
<td>163.6</td>
</tr>
<tr>
<td>2</td>
<td>16.5 cm soft</td>
<td>0.995</td>
<td>2.375</td>
<td>7.02 ( \times 10^4 )</td>
<td>6.76 ( \times 10^8 )</td>
<td>0.965</td>
<td>60.1</td>
<td>63.8</td>
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<td>163.6</td>
</tr>
<tr>
<td>7</td>
<td>15 cm soft + 2 cm bone</td>
<td>0.9015</td>
<td>2.87 ( \times 10^4 )</td>
<td>3.05 ( \times 10^8 )</td>
<td>1</td>
<td>66.3</td>
<td>69.2</td>
<td>66.0</td>
<td>163.6</td>
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</tr>
<tr>
<td>7</td>
<td>17 cm soft</td>
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<td>2.217</td>
<td>6.59 ( \times 10^4 )</td>
<td>6.40 ( \times 10^8 )</td>
<td>1</td>
<td>60.6</td>
<td>64.4</td>
<td>58.6</td>
<td>163.6</td>
</tr>
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</table>
Table 3. Some possible ripple correction factors. Generator types: see table 1. Definitions of $i_{\text{max}}$, $i_{\text{mean}}$, $f_o$, $f_{\text{Kul}}$: see equations (9), (10), (7) and (11). $D_i$ is the ratio of absorbed dose in air to that for constant potential, $f_{\text{IEC}}$ are the $f$ factors of the IEC. (max) and (mean) mean normalisation to unit tube current maximum and mean, respectively. Suffixes: 1: analytical; 2: power function; 3: constant current approximation.

<table>
<thead>
<tr>
<th>Type</th>
<th>$f_{\text{IEC}}$</th>
<th>$i_{\text{mean}} / i_{\text{max}}$</th>
<th>$f_o_{\text{(max)}}$</th>
<th>$f_o_{\text{(mean)}}$</th>
<th>$F_{\text{(max)}}$</th>
<th>$F_{\text{(mean)}}$</th>
<th>$D_{\text{(max)}}$</th>
<th>$D_{\text{(mean)}}$</th>
<th>$F_{\text{Kul(max)}}$</th>
<th>$F_{\text{Kul(mean)}}$</th>
</tr>
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<tbody>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>01</td>
<td>0.74</td>
<td>0.5000</td>
<td>0.4244</td>
<td>0.8488</td>
<td>0.9962</td>
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<td>0.9542</td>
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<td>0.9035</td>
</tr>
<tr>
<td>02</td>
<td>0.74</td>
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<td>0.5718</td>
<td>0.7130</td>
<td>0.4157</td>
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Another factor can be formed from the so-called Kulenkampff formula ($D \sim U^2 i$) giving, approximately, the absorbed dose in air for a slightly filtered beam. The Kulenkampff factor $f_{\text{Kul}}$ can be defined by time averaging of this formula, that is, by

$$f_{\text{Kul}} = \frac{T}{\int_0^T \frac{U^2(t)}{U_{\text{max}}^2} i(t) \, dt}$$

(11)

which expresses the ratio of absorbed dose in air to that of a constant potential beam having the same peak voltage and current, in the $D \sim U^2 i$ approximation.

The ratio of the absorbed dose in air to that for constant potential may also be interesting ($D_i$). In table 3 all these quantities ($f_o$, $f_{\text{Kul}}$, $D_i$ and $F$) are presented for the same current maxima and means. Furthermore, the ratio of the mean to maximum current and factors $f$ of the IEC — where they exist — are also given. All data in table 3 refer to $U = 100$ kVp, 2 mm Al and 1 m of air. (Values of $F$ depend slightly on tube voltage but these differences are always smaller than 1%). In our opinion, the values $F_{\text{(max)}}$ are the best characteristics from the point of view of radiation output; however this question is open to further discussion.

The accuracy of the calculations depends upon the accuracy of the waveforms used, spectra, interaction coefficients, the numerical approximations and the neglected contribution of scattered radiation. The comparison of calculated results with reliable literature data and our own dose measurements shows that the accuracy of the dose calculation (without phantom) is about 6% while that for mean energies is better than 1%. The relative accuracy of the values calculated for different parameters allows three digits to be given, thus even small differences can be evaluated. Figure 2 shows calculated and measured x-ray attenuation curves.

![Figure 2](image_url)

Figure 2. Measured and calculated percentage x-ray transmission for $U = 90$ kV constant potential, 2 mm Al equivalent inherent filtration. Horizontal axis: added filtration in mm Al. ×, measured values with inaccuracies; ○ and full curve, calculated values.
During the calculations, the registration of the intermediate, attenuated x-ray spectra is also possible. Such spectra are shown in figure 3. In figure 3(a) it can be seen how much less the dose of a 2-pulse generator (for the same tube load) is than that of a constant potential one (the areas under the curves are proportional to fluence rates; however, using this the ratio of absorbed doses can also be approximately estimated). Figure 3(b) shows that although the mean energies of the two spectra weighted by fluence are equal, the shapes of the curves are different, i.e. identical spectra cannot be reached by filtration.

Figure 3. Some calculated spectra for $U = 70kV_p$. Nos 1-5: constant potential spectra. 1: 2 mm Al; 2: 2.5 mm Al; 3: 3 mm Al; 4: 4 mm Al; 5: 5 mm Al filtration. Nos 6-7: 2-pulse spectra. 6: 2 mm Al; 7: 4 mm Al filtration, (a) Relative $\phi(E)$ spectra for the same tube current maximum, (b) Spectra Nos 2 and 7 normalised to the same peak value. Mean energies $E_{mean}$ are the same (39.2 keV) but the shapes of the spectra are different.

8. Conclusions

The correct assessment of the performance of new, inverter x-ray generators is an extremely topical problem: we intended to take the first step in the work needed for its resolution.

In addition, the calculation method described has several applications. It can be directly applied in x-ray generator development, for the correct choice of frequency, electrical smoothing etc. The parameters of radiation quantity and quality can be calculated as functions of the supposed waveforms in advance, without realising the machine.

The calculation of the x-ray attenuation for different x-ray generator types can be applied to the choice of radiographic tabletop materials etc, to the planning of radiation shielding, and the estimation of patient dose ratios.

The calculation of the input signal contrast (see table 2) can be applied in detail visibility calculations.

Calculations made using the approximation of spectra by Kramers’ formula show that the value of this approximation is very limited (its accuracy for dose is up to ±40%, and for mean energies it is ±5 keV).

Another variant of the computer program that calculates the effective linear attenuation coefficient as a function of the voltage pulsation can be applied in the evaluation and design of non-invasive tube voltage measuring equipment.

The method opens up possibilities for replacing idealised waveforms with ones measured with a digital storage oscilloscope.

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leader, Mr J Barsai, division manager and Mr F Mustó, head of laboratory, for supporting this work and for fruitful discussions.

Resume

Calculs relatifs à l'ondulation de la tension fournie par les générateurs de rayons x.

Les auteurs présentent des formules permettant le calcul de la dose absorbée, de la fluence de photons et de la fluence énergétique dues à des faisceaux étroits de rayons x produits par des générateurs à tension pulsée et de forme d'onde quelconque. Ils ont écrit un programme d'ordinateur pour déterminer les grandeurs ci-dessus, ainsi que d'autres telles que les énergies moyennes, pour des formes d'onde particulières. Ils suggèrent une nouvelle définition du facteur de correction pour l'ondulation de la tension produite par les générateurs à tension pulsée lorsque l'on teste leurs performances. Ils présentent des résultats numériques et des applications qui font l'objet d'une discussion.

Zusammenfassung

Berechnungen zur Welligkeit der Spannungskurven von Röntgengenerator.


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